CHAPTER 7

ASSIGNMENT PROBLEMS
ปัญหาการจัดงาน

ASSIGNMENT PROBLEMS (1)

- เป้าหมายของการจัดงาน (assignment problem) คือ ต้องการให้เกิดการจัดงานที่ดีที่สุด โดยอาจทำให้เกิดต้นทุนในการดำเนินการที่ต่ำที่สุด หรือ ทำให้เวลาในการทำงานโดยรวมน้อยที่สุด หรือจัดงานให้เกิดผลกำไรโดยรวมต่อองค์กรมากที่สุด ในการจัดงานให้คน หรือเครื่องจักร
- Characteristics of Assignment Problem
  - The number of jobs and the number of people (machines) is equal. (In case of unbalanced, Dummy is used)
  - One job can be assigned to only one machine.
  - One machine can do only one job.
  - Assignment problem can be modeled in Binary LP form.

2

ASSIGNMENT PROBLEMS (2)

- LP Model Formulation:

Indices : i = assignee (i = 1, 2,…,m)

j = task (j = 1, 2,…,n)

Decision Variable:

$$x_{ij} = \begin{cases} 1 & \text{if assignee } i \text{ performs task } j, \\ 0 & \text{if not,} \end{cases}$$

Parameter:

C_{ij} = *the* cost with assignee i performing task j

3

ASSIGNMENT PROBLEMS (3)

- LP Model Formulation (cont.)

Objective: (Minimize the total cost)

$$\text{Minimize} \quad Z = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij},$$

subject to

$$\sum_{j=1}^{n} x_{ij} = 1 \quad \text{for } i = 1, 2, \ldots, n, \qquad \text{(1 Assignee : 1 Task)}$$

$$\sum_{i=1}^{n} x_{ij} = 1 \quad \text{for } j = 1, 2, \ldots, n, \qquad \text{(1 Task : 1 Assignee)}$$

and

$$x_{ij} \geq 0, \quad \text{for all } i \text{ and } j$$
$$(x_{ij} \text{ binary}, \quad \text{for all } i \text{ and } j).$$

4

Assignment Problems (4)

- Algorithm for Assignment Problem: *Hungarian Method*
- Example:

	Job 1	Job 2	Job 3
W 1	10	16	32
W 2	14	22	40
W 3	22	24	34

Assignment Problems (5)

- 1st Step: Row Reduction
  - Select the smallest element from each row and subtract every element in that row.

	Job 1	Job 2	Job 3
W 1	10	16	32
W 2	14	22	40
W 3	22	24	34

→

	Job 1	Job 2	Job 3
W 1	0	6	22
W 2	0	8	26
W 3	0	2	12

Assignment Problems (5)

- 2nd Step: Column Reduction
  - Select the smallest element from each column and subtract every element in that column.

	Job 1	Job 2	Job 3
W 1	0	6	22
W 2	0	8	26
W 3	0	2	12

→

	Job 1	Job 2	Job 3
W 1	0	4	10
W 2	0	6	14
W 3	0	0	0

Assignment Problems (6)

- 3rd Step: Check Optimality
  - Draw the line pass the cell that has 0 by using minimum no. of line.

	Job 1	Job 2	Job 3
W 1	0	4	10
W 2	0	6	14
W 3	0	0	0

So, 2 lines ≠ 3 rows

Not Optimal!!

  - Solution is optimal when

***No. of line = No. of row ***

Assignment Problems (7)

- 4th Step: Improving the solution

	Job 1	Job 2	Job 3
W 1	0	4	10
W 2	0	6	14
W 3	0	0	0

So, 3 lines = 3 rows

Optimal!!

	Job 1	Job 2	Job 3
W 1	0	0	6
W 2	0	2	10
W 3	4	0	0

- สำหรับตัวเลขที่ไม่โดนเส้นตัดผ่าน ให้เลือกตัวเลขที่น้อยที่สุด แล้วนำไปหักล้างกับตัวเลขชุดที่ไม่โดนเส้นลากผ่าน แล้วนำตัวเลขนั้นไปเพิ่มที่จุดตัดของเส้นที่ลากผ่านค่าศูนย์ (0)
- ลากเส้นผ่านตำแหน่งที่เกิดเลขศูนย์ (0) อีกครั้ง
- Solution is optimal when

***No. of line = No. of row ***

Assignment Problems (8)

- Example:

	Job 1	Job 2	Job 3
W 1	10	16	32
W 2	14	22	40
W 3	22	24	34

- Solution: Z = 14 + 16 + 34 = 64
- When, Worker 1 – Job 2, Worker 2 – Job 1 and Worker 3 – Job 3

Additional with unacceptable cases

- When additional condition is added;
  - Min Problem: Assign “M” Cost
  - Max Problem: Assign “- M” Profit

Example for Unacceptable Case (1)

- Page 335: Job Shop Company

TABLE 8.24 Materials-handling cost data ($) for Job Shop Co.

		Location 1	2	3	4
Machine	1	13	16	12	11
	2	15	—	13	20
	3	5	7	10	6

TABLE 8.25 Cost table for the Job Shop Co. assignment problem

		Task (Location) 1	2	3	4
Assignee (Machine)	1	13	16	12	11
	2	15	*M*	13	20
	3	5	7	10	6
	4(*D*)	0	0	0	0

Example for Unacceptable Case (2)

- Initial Table:

	1	2	3	4
1	13	16	12	11
2	15	M	13	20
3	5	7	10	6
4	0	0	0	0

- Row Reduction:

	1	2	3	4
1				
2				
3				
4				

- Column Reduction:

	1	2	3	4
1				
2				
3				
4				

Example for Unacceptable Case (3)

- Draw Line

	1	2	3	4
1				
2				
3				
4				

	1	2	3	4
1				
2				
3				
4				

- Check Optimality: No. of line = No. of Row
- Optimal Solution:
  - Total Cost (Z) =
  - When,

Transportation to Assignment (1)

- Page 339: Assigning Product to Plant

TABLE 8.27 Data for the Better Products Co. problem

		Unit Cost ($) for Product				Capacity Available
		1	2	3	4	
Plant	1	41	27	28	24	75
	2	40	29	—	23	75
	3	37	30	27	21	45
Production rate		20	30	30	40	

- *Option 1:* Permit product splitting, where the same product is produced in more than one plant.
  - Transportation Problem
- *Option 2:* Prohibit product splitting.
  - Assignment Problem

Transportation to Assignment (2)

	Product					Supply
Plant	1	2	3	4	5(D)	
1	41	27	28	24	0	75
2	40	29	M	23	0	75
3	37	30	27	31	0	45
Demand	20	30	30	40	75	

	Product				
Plant	1	2	3	4	5(D)
1a	820	810	840	960	0
1b	820	810	840	960	0
2a	800	870	M	920	0
2b	800	870	M	920	0
3	740	900	810	840	M

Additional with Max Problem

- The methods that can be used to solved transportation problems and assignment problems are talk about only when the problem is Min problem.
- So, how to handle Max problem with the same procedure??

Max Assignment Problem (1)

- Obtain an ***equivalent minimization assignment problem*** by converting all the elements in the matrix to ***opportunity losses*** or ***regret table.***
  - Subtracting all element in each column from the largest element in that column.
- Apply Hungarian Method on the regret table

Max Assignment Problem (2)

- Example: This is the profit table (in thousand dollars) solve this problem as assignment problem.

	Job 1	Job 2	Job 3	Job 4
W 1	81	95	78	22
W 2	85	102	77	33
W 3	81	103	79	26
W4	75	98	76	26

Max Assignment Problem (3)

- Generate Regret Table: Subtracting every element in each column with the largest elements in each column

	Job 1	Job 2	Job 3	Job 4
W 1	81	95	78	22
W 2	**85**	102	77	**33**
W 3	81	**103**	**79**	26
W4	75	98	76	26

	Job 1	Job 2	Job 3	Job 4
W 1	85-81	103-95	79-78	33-22
W 2	85-85	103-102	79-77	33-33
W 3	85-81	103-103	79-79	33-26
W4	85-75	103-98	79-76	33-26

	Job 1	Job 2	Job 3	Job 4
W 1	4	8	1	11
W 2	0	1	2	0
W 3	4	0	0	7
W4	10	5	3	7

Regret Table !!!

MAX ASSIGNMENT PROBLEM (4)

- Apply Hungarian Algorithm on Regret Table
  - Row reduction:

	Job 1	Job 2	Job 3	Job 4
W 1	4	8	**1**	11
W 2	**0**	1	2	**0**
W 3	4	**0**	**0**	7
W4	10	5	**3**	7

	Job 1	Job 2	Job 3	Job 4
W 1	3	7	0	10
W 2	0	1	2	0
W 3	4	0	0	7
W4	7	2	0	4

  - Column Reduction:

	Job 1	Job 2	Job 3	Job 4
W 1	3	7	0	10
W 2	0	1	2	0
W 3	4	0	0	7
W4	7	2	0	4

MAX ASSIGNMENT PROBLEM (5)

- Apply Hungarian Algorithm on Regret Table
  - Draw minimum no. of line:

	Job 1	Job 2	Job 3	Job 4
W 1	3	7	0	10
W 2	0	1	2	0
W 3	4	0	0	7
W4	7	2	0	4

  - Optimality Test: **no. of lines = no. of rows**
    - No. of lines (3) ≠ No. of rows (4) ➔ **Not Optimal !!**

MAX ASSIGNMENT PROBLEM (6)

- Apply Hungarian Algorithm on Regret Table
  - Improve Solution:

	Job 1	Job 2	Job 3	Job 4
W 1	**3**	7	0	10
W 2	0	1	2	0
W 3	4	0	0	7
W4	7	2	0	4

	Job 1	Job 2	Job 3	Job 4
W 1	0	7	0	7
W 2	0	4	5	0
W 3	1	0	0	4
W4	4	2	0	1

MAX ASSIGNMENT PROBLEM (7)

- Apply Hungarian Algorithm on Regret Table
  - Draw Lines:

	Job 1	Job 2	Job 3	Job 4
W 1	**0**	7	0	7
W 2	0	4	5	**0**
W 3	1	**0**	0	4
W4	4	2	**0**	1

  - Optimality Test: **no. of lines = no. of rows**
    - No. of lines (4) = No. of rows (4) ➔ **Optimal !!**

MAX ASSIGNMENT PROBLEM (8)

- Optimal Solution:

	Job 1	Job 2	Job 3	Job 4
W 1	**0**	7	0	7
W 2	0	4	5	**0**
W 3	1	**0**	0	4
W4	4	2	**0**	1

	Job 1	Job 2	Job 3	Job 4
W 1	**81**	95	78	22
W 2	85	102	77	**33**
W 3	81	**103**	79	26
W4	75	98	**76**	26

- Max Profit Z = 81+ 103 + 76 + 33 = 293 (in thousand dollars)
- When, Worker 1 – Job 1, Worker 2 – Job 4, Worker 3 – Job 4 and Worker 4 – Job 3

Q&A