

CHAPTER 7

ASSIGNMENT PROBLEMS ปัญหาการจัดงาน



Original work by K.Yaibuathet August, 2009

ASSIGNMENT PROBLEMS (1)

- o เป้าหมายของการจัดงาน (assignment problem) คือ ต้องการให้ เกิดการจัดงานที่ดีที่สด โดยอาจทำให้เกิดต้นทนในการดำเนินการที่ ต่ำที่สด หรือ ทำให้เวลาในการทำงานโดยรวมน้อยที่สด หรือจัดงาน ให้เกิดผลกำไรโดยรวมต่อองค์กรมากที่สด ในการจัดงานให้คน หรือ เครื่องจักร
- Characteristics of Assignment Problem
 - The number of jobs and the number of people (machines) is equal. (In case of unbalanced, Dummy is used)
 - One job can be assigned to only one machine.
 - One machine can do only one job.
 - Assignment problem can be modeled in Binary LP form.

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ASSIGNMENT PROBLEMS (2)

LP Model Formulation:

Indices:

i = assignee (*i* = 1, 2,...,m)

j = task

(i=1, 2, ..., n)

Decision Variable:

 $x_{ij} = \begin{cases} 1 & \text{if assignee } i \text{ performs task } j, \\ 0 & \text{if not,} \end{cases}$

Parameter:

 C_{ii} = the cost with assignee *i* performing task *j*

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ASSIGNMENT PROBLEMS (3)

o LP Model Formulation (cont.)

Objective: (Minimize the total cost)

Minimize
$$Z = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij},$$

subject to

 $\sum_{j=1}^{n} x_{ij} = 1$ for i = 1, 2, ..., n, (1 Assignee : 1 Task)

 $\sum_{i=1}^{n} x_{ij} = 1$ for j = 1, 2, ..., n, (1 Task : 1 Assignee)

 $x_{ij} \ge 0$, for all i and j $(x_{ij} \text{ binary}, \text{ for all } i \text{ and } j).$

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ASSIGNMENT PROBLEMS (4)

- o Algorithm for Assignment Problem: Hungarian Method
- Example:

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	Job 1	Job 2	Job 3
W 1	10	16	32
W 2	14	22	40
W 3	22	24	34

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ASSIGNMENT PROBLEMS (5)

- 1st Step: Row Reduction
 - Select the smallest element from each row and subtract every element in that row.

	Job 1	Job 2	Job 3
W 1	(10)	16	32
W 2	(14)	22	40
W 3	(22)	24	34

	Job 1	Job 2	Job 3
W 1	0	6	22
W 2	0	8	26
W 3	0	2	12

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ASSIGNMENT PROBLEMS (5)

- 2nd Step: Column Reduction
 - Select the smallest element from each column and subtract every element in that column.

	Job 1	Job 2	Job 3
W 1	0	6	22
W 2	0	8	26
W 3	(0)	(2)	(12)

	Job 1	Job 2	Job 3
W 1	0	4	10
W 2	0	6	14
W 3	0	0	0

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ASSIGNMENT PROBLEMS (6)

- 3rd Step: Check Optimality
 - Draw the line pass the cell that has 0 by using minimum no. of line.

	Job	1	Job 2	Job 3
W 1	0		4	10
W 2	0		6	14
W 3 -			0	0
VV 3 -	ľ		Ü	Ü

So, 2 lines ≠ 3 rows

Not Optimal!!

Solution is optimal when

***No. of line = No. of row ***

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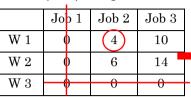
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ASSIGNMENT PROBLEMS (7)

• 4th Step: Improving the solution



So, 3 lines = 3 rows

Optimal!!

	Job 1	Job 2	Job 3
W 1	0		6
W 2	1	2	10
W 3	4	0	

- สำหรับตัวเลขที่ไม่โดนเส้นตัดผ่าน ให้เลือกตัวเลขที่น้อยที่สุด แล้วนำไปหักล้างกับตัวเลขชุดที่ไม่โดนเส้นลากผ่าน แล้วนำ ตัวเลขนั้นไปเพิ่มที่จุดตัดของเส้นที่ลากผ่านค่าศูนย์ (0)
- ลากเส้นผ่านตำแหน่งที่เกิดเลขศูนย์ (0) อีกครั้ง
- Solution is optimal when

***No. of line = No. of row ***

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ASSIGNMENT PROBLEMS (8)

• Example:

	Job 1	Job 2	Job 3
W 1	10	(16)	32
W 2	(14)	22	40
W 3	22	24	(34)

- o Solution: Z = 14 + 16 + 34 = 64
- When, Worker 1 Job 2, Worker 2 Job 1 and Worker 3 Job 3

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ADDITIONAL WITH UNACCEPTABLE CASES

- When additional condition is added;
 - Min Problem: Assign "M" Cost
 - Max Problem: Assign "- M" Profit

Example for Unacceptable Case (1)

o Page 335: Job Shop Company

TABLE 8.24 Materials-handling cost data (\$) for Job Shop Co.

	(1)						
			Location				
		1	2	3	4		
	1	13	16	12	11		
Machine	2	15	_	13	20		
	3	5	7	10	6		

TABLE 8.25 Cost table for the Job Shop Co. assignment problem

		Task (Location)			
		1	2	3	4
	1	13	16	12	11
Assignee	2	15	М	13	20
(Machine)	3	5	7	10	6
	4(D)	0	0	0	0

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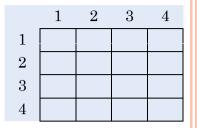
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EXAMPLE FOR UNACCEPTABLE CASE (2)

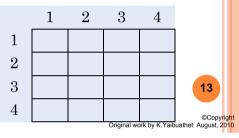
Initial Table:

	1	2	3	4
1	13	16	12	11
2	15	M	13	20
3	5	7	10	6
4	0	0	0	0

- Row Reduction:



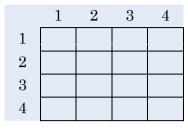
Column Reduction:

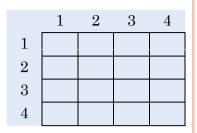


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EXAMPLE FOR UNACCEPTABLE CASE (3)

Draw Line





- Check Optimality: No. of line = No. of Row
- Optimal Solution:
 - o Total Cost (Z) =
 - When,

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Transportation to Assignment (1)

o Page 339: Assigning Product to Plant

TABLE 8.27 Data for the Better Products Co. problem

			Unit Cost (\$)	for Product		Cin-
		1	2	3	4	Capacity Available
	1	41	27	28	24	75
Plant	2	40	29	_	23	75
	3	37	30	27	21	45
Production	rate	20	30	30	40	

- Option 1: Permit product splitting, where the same product is produced in more than one plant.
 - Transportation Problem
- Option 2: Prohibit product splitting.
 - Assignment Problem

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Transportation to Assignment (2)

		Product					
Plant	1	2	3	4	5(D)		
1	41	27	28	24	0	75	
2	40	29	\mathbf{M}	23	0	75	
3	37	30	27	31	0	45	
Demand	20	30	30	40	75		

	Product					
Plant	1	2	3	4	5(D)	
1a	820	810	840	960	0	
1b	820	810	840	960	0	
2a	800	870	\mathbf{M}	920	0	
2b	800	870	\mathbf{M}	920	0	
3	740	900	810	840	\mathbf{M}	

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ADDITIONAL WITH MAX PROBLEM

- The methods that can be used to solved transportation problems and assignment problems are talk about only when the problem is Min problem.
- o So, how to handle Max problem with the same procedure??

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MAX ASSIGNMENT PROBLEM (1)

- Obtain an equivalent minimization assignment problem by converting all the elements in the matrix to opportunity losses or regret table.
 - Subtracting all element in each column from the largest element in that column.
- Apply Hungarian Method on the regret table

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MAX ASSIGNMENT PROBLEM (2)

• Example: This is the profit table (in thousand dollars) solve this problem as assignment problem.

	Job 1	Job 2	Job 3	Job 4
W 1	81	95	78	22
W 2	85	102	77	33
W 3	81	103	79	26
W4	75	98	76	26

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Regret Table !!!

MAX ASSIGNMENT PROBLEM (3)

 Generate Regret Table: Subtracting every element in each column with the largest elements in each column

	Job 1	Job 2	Job 3	Job 4
W 1	81	95	78	22
W 2	85	102	77	33
W 3	81	103	79	26
W4	75	98	76	26

	Job 1	Job 2	Job 3	Job 4
W 1	85-81	103-95	79-78	33-22
W 2	85-85	103-102	79-77	33-33
W 3	85-81	103-103	79-79	33-26
W4	85-75	103-98	79-76	33-26

	Job 1	Job 2	Job 3	Job 4
W 1	4	8	1	11
W 2	0	1	2	0
W 3	4	0	0	7
WA	10	5	3	7

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MAX ASSIGNMENT PROBLEM (4)

- Apply Hungarian Algorithm on Regret Table
 - Row reduction:

	Job 1	Job 2	Job 3	Job 4
W 1	4	8	1	11
W 2	0	1	2	0
W 3	4	0	0	7
W4	10	5	3	7

	Job 1	Job 2	Job 3	Job 4
W 1	3	7	0	10
W 2	0	1	2	0
W 3	4	0	0	7
W4	7	2	0	4

Column Reduction:

	Job 1	Job 2	Job 3	Job 4
W 1	3	7	0	10
W 2	0	1	2	0
W 3	4	0	0	7
W4	7	2	0	4

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MAX ASSIGNMENT PROBLEM (5)

- Apply Hungarian Algorithm on Regret Table
 - Draw minimum no. of line:

	Job 1	Jo	2	Jo	o 3	Job 4
W 1	3	,	7			10
W 2	0		-	- 1		0
W 3	4)	()	7
W4	7	2	2	()	4

- Optimality Test: no. of lines = no. of rows
 - o No. of lines (3) ≠ No. of rows (4) → Not Optimal !!

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MAX ASSIGNMENT PROBLEM (6)

- o Apply Hungarian Algorithm on Regret Table
 - Improve Solution:

	•					
	Job 1	Jol	2	Jo	o 3	Job 4
W 1	3	,	7	()	10
W 2	0	-		,)	0
	,					
W 3	4	()	()	7
W4	7	2	}	()	4

			Job 1	Job 2	Job 3	Job 4
		W 1	0	7	0	7
1	_	W 2	0	4	5	0
		W 3	1	0	0	4
		W4	4	2	0	1

MAX ASSIGNMENT PROBLEM (7)

- Apply Hungarian Algorithm on Regret Table
 - Draw Lines:

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		Job 1	Jok	2	Jol	3 (Job 4
_	W 1	0	7		(7
	W o	0	1				0
	W2	1			,		4
	W 3	1	U		·		4
	W4	4	2		(1

• Optimality Test: no. of lines = no. of rows o No. of lines (4) = No. of rows (4) → Optimal !!

MAX ASSIGNMENT PROBLEM (8)

Optimal Solution:

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	Job 1	Job 2	Job 3	Job 4
W 1	0	7	0	7
W 2	0	4	5	0
W 3	1	0	0	4
W4	4	2	0	1

	Job 1	Job 2	Job 3	Job 4
W 1	81	95	78	22
W 2	85	102	77	33
W 3	81	103	79	26
W4	75	98	76	26

- o Max Profit Z = 81 + 103 + 76 + 33 = 293 (in thousand dollars)
- o When, Worker 1 Job 1, Worker 2 Job 4, Worker 3 Job 4 and Worker 4 – Job 3



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